C1 MAY 2015

1. Simplify

(a) $(2\sqrt{5})^2$ **(1)**

(b) $\dfrac{\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}$ giving your answer in the form $a + \sqrt{b}$, where a and b are integers. **(4)**

a) $(2\sqrt{5})^2 = 2 \times \sqrt{5} \times 2 \times \sqrt{5} = 4 \times 5 = \underline{20}$

b) $\dfrac{\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}} \dfrac{\times (2\sqrt{5} + 3\sqrt{2})}{\times (2\sqrt{5} + 3\sqrt{2})} = \dfrac{2\sqrt{10} + 3 \times 2}{20 - 18}$

$(3\sqrt{2})^2 = 9 \times 2 = 18$ $\qquad = \dfrac{2\sqrt{10} + 6}{2} = \underline{\sqrt{10} + 3}$

2. Solve the simultaneous equations

$$y - 2x - 4 = 0$$

$$4x^2 + y^2 + 20x = 0$$

(7)

$y = 2x + 4$

$y^2 = (2x+4)(2x+4) = 4x^2 + 16x + 16$

$4x^2 + (4x^2 + 16x + 16) + 20x = 0$

$8x^2 + 36x + 16 = 0$ $\div 4$ $2x^2 + 9x + 4 = 0$

$(2x+1)(x+4) = 0$ $x = -\frac{1}{2}$ $y = 3$

$x = -4$ $y = \cancel{5}\ -4$

$(-\frac{1}{2}, 3)$; $\cancel{(-4, -5)}$

$(-4, -4)$

3. Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\frac{dy}{dx}$ **(3)**

(b) $\int y \, dx$ **(3)**

$y = 4x^3 - 5x^{-2}$

a) $\frac{dy}{dx} = 12x^2 + 10x^{-3} = 12x^2 + \frac{10}{x^3}$

b) $\int y dx = \frac{4x^4}{4} - \frac{5x^{-1}}{-1} + c = x^4 + \frac{5}{x} + c$

4. (i) A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geqslant 1$$

$$U_1 = 4 \text{ and } U_2 = 4$$

Find the value of

(a) U_3

(1)

(b) $\sum_{n=1}^{20} U_n$

(2)

(ii) Another sequence $V_1, V_2, V_3, \ldots$ is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geqslant 1$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant}$$

(a) Find V_3 and V_4 in terms of k.

(2)

Given that $\sum_{n=1}^{5} V_n = 165$,

(b) find the value of k.

(3)

a) $U_3 = 2U_2 - U_1 = 2(4) - 4 \quad = \underline{4}$

b) $\sum_{n>1}^{20} U_n = 4 \times 20 = \underline{80}$

(ii) $V_3 = 2V_2 - V_1 = 2(2k) - k = 3k$

$V_4 = 2V_3 - V_2 = 2(3k) - 2k = 4k$

$V_5 = 2V_4 - V_3 = 2(4k) - 3k = 5k$

b) $\sum_{1}^{5} V_n = k + 2k + 3k + 4k + 5k = 15k = 165$

$\underline{k = 11}$

5. The equation

$$(p-1)x^2 + 4x + (p-5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p.

(4)

no real roots $\Rightarrow b^2 - 4ac < 0$

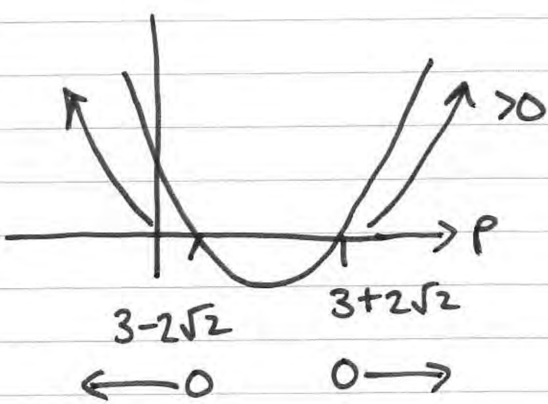
$4^2 - 4(p-1)(p-5) < 0$

$16 - 4p^2 + 24p - 20 < 0 \Rightarrow 4p^2 - 24p + 4 > 0$

$\div 4$ $\quad p^2 - 6p + 1 > 0 \Rightarrow (p-3)^2 - 9 + 1 > 0$

$(p-3)^2 - 8 > 0$

$(p-3)^2 - 8 = 0 \Rightarrow (p-3)^2 = 8 \Rightarrow p - 3 = \pm\sqrt{8} \Rightarrow p = 3 \pm 2\sqrt{2}$



$p > 3 + 2\sqrt{2}$

or

$p < 3 - 2\sqrt{2}$

6. The curve C has equation

$$y = \frac{(x^2+4)(x-3)}{2x}, \quad x \neq 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form. **(5)**

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

$$y = \frac{x^3 - 3x^2 + 4x - 12}{2x} = \frac{1}{2}x^2 - \frac{3}{2}x + 2 - 6x^{-1}$$

$$\frac{dy}{dx} = x - \frac{3}{2} + \frac{6}{x^2}$$

$$x = -1 \quad y = \frac{(1+4)(-1-3)}{-2} = \frac{5 \times -4}{-2} = 10$$

$$M_t = -1 - \frac{3}{2} + \frac{6}{1} = \frac{7}{2}$$

$$y - y_1 = m(x - x_1) \qquad y - 10 = \frac{7}{2}(x+1)$$

$$2y - 20 = 7x + 7$$

$$\Rightarrow 7x - 2y + 27 = 0$$

7. Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

a) $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

b) $8y^2 - 9y + 1 = 0$

$(8y-1)(y-1) = 0$

$y = \frac{1}{8} \quad y = 1$

$2^x = \frac{1}{8} = \frac{1}{2^3} = 2^{-3} \quad \therefore \underline{x = -3}$

$2^x = 1 \quad \therefore \underline{x = 0}$

8. (a) Factorise completely $9x - 4x^3$

(3)

(b) Sketch the curve C with equation

$$y = 9x - 4x^3$$

Show on your sketch the coordinates at which the curve meets the x-axis.

(3)

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

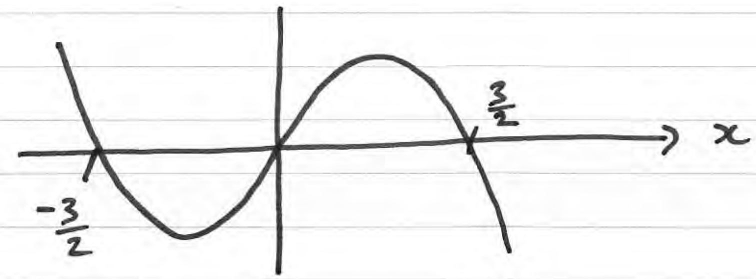
(c) Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found.

(4)

a) $x(9-4x^2) = x(3-2x)(3+2x)$

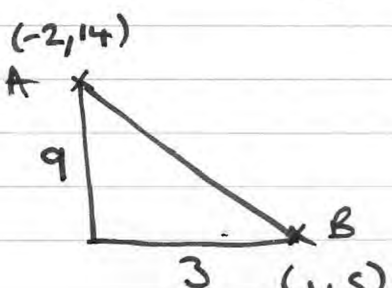
b) $y = x(3-2x)(3+2x)$ } hell $\ddot{n}$ $\underline{-4x^3}$

$0,\ \frac{3}{2},\ -\frac{3}{2}$



c) $x = -2 \quad y = 9(-2) - 4(-2)^3 = -18 + 32 = 14$

$x = 1 \quad y = 9(1) - 4(1)^3 = 5$



$AB^2 = 9^2 + 3^2 = 81 + 9 = 90$

$AB = \sqrt{90} = \sqrt{9}\sqrt{10} = 3\sqrt{10}$

$\therefore \underline{k = 3}$

9. Jess started work 20 years ago. In year 1 her annual salary was £17000. Her annual salary increased by £1500 each year, so that her annual salary in year 2 was £18500, in year 3 it was £20000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32000 in year k. Her annual salary then remained at £32000.

(a) Find the value of the constant k. **(2)**

(b) Calculate the total amount that Jess has earned in the 20 years. **(5)**

$u_1 = a = 17000 \quad d = 1500$
$u_2 = 18500$

$u_n = 32000 = a + (n-1)d = 17000 + (n-1) \times 1500$

$\Rightarrow 15000 = 1500(n-1) \Rightarrow n - 1 = 10 \quad \therefore n = 11$

$\therefore u_{11} = 32000 \quad \therefore \underline{k = 11}$

$S_{11} = \frac{1}{2}n(a + L) = \frac{11}{2}(17000 + 32000)$

$= \frac{11}{2} \times 49000 \quad 49 \times 11 = 539$
$\div 2 = 269.5$

$= 269500$

9 years $\times 32000 = 288000$
$+ 269500$
$\underline{557500}$ $\quad$ $\underline{£557500}$

alt $u_{10} = 30500$

$S_{10} = \frac{1}{2}(10)(17000 + 30500) = 5 \times 47500$
$= 237500$
$+ 10 \times 32000 \quad + 320000$
$\underline{557500}$

10. A curve with equation $y = f(x)$ passes through the point (4, 9).

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

(a) find f(x), giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$

(b) Find the x coordinate of P.

(5)

a) $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{1}{2}} + 2$

$f(x) = \frac{\frac{3}{2}x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{\frac{9}{4}x^{\frac{1}{2}}}{\frac{1}{2}} + 2x + c$

$f(x) = x^{\frac{3}{2}} - \frac{9}{2}x^{\frac{1}{2}} + 2x + c$

$(4,9) \Rightarrow 9 = (\sqrt{4})^3 - \frac{9}{2}\sqrt{4} + 8 + c \Rightarrow 9 = 8 - 9 + 8 + c$

$\therefore \underline{c = 2}$

$\therefore f(x) = x^{\frac{3}{2}} - \frac{9}{2}x^{\frac{1}{2}} + 2x + 2$

b) $2y + x = 0 \Rightarrow y = -\frac{1}{2}x \quad \therefore M_n = -\frac{1}{2} \quad \therefore M_t = 2 = f'(x)$

$2 = \frac{3}{2}\sqrt{x} - \frac{9}{4\sqrt{x}} + 2 \quad \therefore \frac{3}{2}\sqrt{x} - \frac{9}{4\sqrt{x}} = 0$

$\Rightarrow \frac{3}{2}\sqrt{x} = \frac{9}{4\sqrt{x}} \Rightarrow \frac{12x}{2} = 9 \Rightarrow 6x = 9$

$\therefore \underline{x = 1.5}$